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Performance of Maximum Likelihood, Ridge, and Generalized Ridge Estimators in Handling Multicollinearity in Poisson Inverse Gaussian Regression

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ABSTRACT: Poisson Inverse Gaussian Regression Model is a commonly used method for analyzing data that experiences overdispersion. Poisson Inverse Gaussian Regression typicall use Maximum Likelihood Estimator (MLE) parameter estimation, by fulfilling one of its assumptions wich is multicollinearity. With the existence of multicollinearity can cause the results of regression method analysis is not accurate because the results of the analysis are unstable and biased. This research uses data on the number of crimes reported in East Java Province in 2024, but the data experiences multicollinearity. To overcome multicollinearity, several methods can be used, that is PIGRRE and PIGGRE. The PIGRRE method works by adding a penalty to the magnitude the regression coefficient so that it can reduce the estimation variance of the estimate without increasing the bias. The PIGGRE method applies a diagonal shrinkage matrix that assigns a different shrinkage value to each predictor independent variable. The results of this study show that the best method for overcoming multicollinearity is the method with the smallest MSE value, namely the PIGGRE method with a ridge k_2 parameter of 0.1974.

KEYWORDS: Poisson Inverse Gaussian, PIGMLE, PIGGRE, PIGGGRE, Mean Square Error

I. INTRODUCTION

The Poisson Inverse Gaussian Regression (PIGR) model is a widely used method for analyzing count data that exhibit overdispersion. In PIGR, parameter estimation is typically performed using the Maximum Likelihood Estimator (MLE). However, the reliability of MLE in this model is often compromised by the presence of multicollinearity among explanatory variables (Almulhim et al., 2025). Multicollinearity can lead to unstable parameter estimates and inflated variances, resulting in biased and unreliable regression results.To address this issue, several alternative estimation approaches have been proposed, including the Poisson Inverse Gaussian Ridge Regression Estimator (PIGRRE) and the Poisson Inverse Gaussian Generalized Ridge Estimator (PIGGRE). In PIGRRE, a penalty term is added to the magnitude of the regression coefficients, which helps reduce estimation variance without substantially increasing bias (Hoerl & Kennard, 1970). The Generalized Ridge Estimator is an extension of the ridge regression method introduced by Hoerl and Kennard (1970). It employs a diagonal shrinkage matrix that allows each explanatory variable to have a different shrinkage parameter, providing greater flexibility in handling multicollinearity.The Generalized Ridge Estimator is expected to yield more stable estimation results than both MLE and the conventional Ridge Regression Estimator when multicollinearity is present. This study uses data on the number of crimes reported in East Java Province in 2024, which are suspected to exhibit both overdispersion and multicollinearity. Therefore, the aim of this research is to compare the performance of PIGMLE, PIGRRE, and PIGGRE in addressing multicollinearity and to determine the best method based on the Mean Squared Error (MSE) criterion.

II. LITERATURE REVIEW

Poisson Inverse Gaussian Maximum Likelihood Estimator

Parameter estimation in the Poisson Inverse Gaussian model is typically performed using the Maximum Likelihood Estimator (MLE), but it is sensitive to multicollinearity (Farghali et al., 2025). The process of obtaining the MLE estimate for the Poisson Inverse Gaussian begins with forming the log-likelihood function using the formula:

$$\ell(\mu_i,\phi)=\sum_{i=1}^{n}\left[y_i\log(\mu_i)+\phi-\log(y_i!)+\frac{1}{2}\log\left(\frac{1}{\phi}\right)-\frac{2y_i-1}{4}log\left(1+2\frac{\mu_i}{\phi}\right)+logR_{y-0.5\phi}\left(\phi\sqrt{1+2\frac{\mu_i}{\phi}}\right)\right]$$

The log-likelihood function with respect to the regression parameter β becomes:

$$\ell(\beta) = \sum_{i=1}^{n}\left[y_i x_i^T\beta + \phi - \log(y_i!) + \frac{1}{2}\log\left(\frac{1}{\phi}\right) - \frac{2y_i-1}{4} log\left(1+2\frac{\exp(x_i^T\beta)}{\phi}\right)\right.$$
$$\left. + \quad logR_{y-0.5\phi}\left(\phi\sqrt{1+2\frac{\exp(x_i^T\beta)}{\phi}}\right)\right]$$

The Maximum Likelihood Estimator (MLE) is obtained by differentiating the log-likelihood function with respect to each parameter and setting the resulting derivatives equal to zero. The first derivative with respect to the regression parameter is expressed as follows:

$$\frac{\partial\ell}{\partial\beta_j} = \sum_{i=1}^{n}\left[x_i\left(\mathrm{y}_i - \frac{R_{y-0.5\phi}(\alpha)}{\sqrt{1+\frac{2}{\phi}\exp(x_i^T\beta)}}\exp(x_i^T\beta)\right)\right] = 0$$

Meanwhile, the first derivative with respect to the dispersion parameter (φ) is expressed as:

$$\frac{\partial\ell}{\partial\phi} = \sum_{i=1}^{n}\left[-\phi^2 - \mathrm{y}_i\phi - \frac{\phi^2 R_{y-0.5\phi}(\alpha)\left(1+\frac{\exp(x_i^T\beta)}{\phi}\right)}{\sqrt{1+\frac{2}{\phi}\exp(x_i^T\beta)}}\right] = 0$$

To estimate the parameters β and φ through MLE, the Newton-Raphson algorithm is used because the equation is nonlinear. After the iterative process is complete, the Maximum Likelihood Estimator (MLE) parameter estimator for the Inverse Gaussian Poisson model can be expressed as:

$$\hat{\beta}_{MLE} = \left(X^T\widehat{W}X\right)^{-1}X^T\widehat{W}\hat{u}$$

To evaluate the accuracy of the estimation, the Mean Square Error Matrix (MSEM) and the Mean Square Error (MSE) for $\hat{\beta}_{MLE}$ are determined thru the spectral decomposition of the matrix $X^T\widehat{W}X = QGQ^T$. Where $G_K = diag(g_1+k, g_2+k, \dots, g_p+k)$ is the diagonal matrix of the eigenvalues of $X^T\widehat{W}X$ and Q is the orthogonal matrix with columns corresponding to the eigenvectors of $X^T\widehat{W}X$. And for the formula for Mean Squared Error (MSE):

$$MSE\left(\hat{\beta}_{MLE}\right) = \hat{\phi}\sum_{j=1}^{p}\frac{1}{g_j}$$

where $\hat{\phi}$ is the estimated value of ϕ and is calculated as $\hat{\phi} = \frac{\Sigma_{j=1}^{n}\frac{(y_i+\hat{\mu}_i)^2}{V(\hat{\mu}_i)}}{n-p}$

Poisson Inverse Gaussian Ridge Regression Estimator

Poisson Inverse Gaussian Regression the developed estimator can address multicollinearity in Poisson Inverse Gaussian Regression (Batool, et al., 2023). This estimator adds a penalty parameter k, resulting in more stable results compared to the MLE method. The general form of the PIGRRE estimator is expressed as follows:

$$\hat{\beta}_k = \left(X^T\widehat{W}X + kI\right)^{-1}X^T\widehat{W}X\hat{\beta}_{MLE}$$

where k > 0 represents the PIGRRE parameter, and I is the $p \times p$ identity matrix. . And for the formula for Mean Squared Error (MSE) :

$$MSE\left(\hat{\beta}_k\right) = \hat{\phi}\sum_{j=1}^{p}\frac{g_j}{\left(g_j+k\right)^2} + \sum_{j=1}^{p}\frac{k^2\alpha_j^2}{\left(g_j+k\right)^2}$$

where α_j represents the j-th component of the vector $\alpha = Q^T \hat{\beta}_{MLE}$, which represents the values of the coefficients fitted using Maximum Likelihood Estimator. The ridge parameter k can play a role in reducing multicollinearity and preventing overfitting. The ridge parameter k can play a role in reducing multicollinearity and preventing overfitting.

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$$k_1 = min\left(\frac{\hat{\phi}}{2\hat{\alpha}_j^2 + \frac{\hat{\phi}}{g_j}}\right), k_2 = min\left(\frac{\hat{\phi}}{\hat{\alpha}_j^2}\right), k_3 = \left(\frac{\hat{\phi}}{\sum_j^p \hat{\alpha}_j^2}\right), k_4 = \prod_j^p \left(\frac{\hat{\phi}}{2\hat{\alpha}_j^2 + \frac{\hat{\phi}}{g_j}}\right)^{\frac{1}{p}}, k_5 = p\sum_{j=1}^{p} \frac{\hat{\phi}}{2\hat{\alpha}_j^2 + \frac{\hat{\phi}}{g_j}}$$

This alternative ridge k value can be used to obtain more stable parameter estimates in the Poisson Inverse Gaussian model if there is multicollinearity.

Poisson Inverse Gaussian Generalized Ridge Estimator

The Generalized Ridge Estimator was first developed by Loesgen (1990) as an extension of the Ridge Regression Estimator, applying a different degree of shrinkage to each variable. The general form of the PIGGRE estimate is expressed as follows :

$$\hat{\beta}_k = \left(X^T \widehat{W} X + K\right)^{-1} X^T y = \left(X^T \widehat{W} X + K\right)^{-1} X^T \widehat{W} X \hat{\beta}_{MLE}$$

And the MSE formula for PIGGRE is:

$$MSE(\hat{\beta}_k) = \hat{\phi}\sum_{j=1}^{p} \frac{g_j}{(g_j + k_j)^2} + \sum_{j=1}^{p} \frac{k_j^2 \alpha_j^2}{(g_j + k_j)^2}$$

In the Poisson Inverse Gaussian Generalized Ridge Estimator(PIGGRE), the selection of the ridge parameter k is important for determining the amount of shrinkage in each regression coefficient. According to Almulhim, et al. (2025), adjusted to the modified general form of ridge, the Generalized Ridge Estimator parameter can be defined in several alternative forms as follows:

$$k_1 = diag\left(\frac{\hat{\phi}}{\hat{\alpha}_j^2}\right), k_2 = diag\left(\frac{1}{\hat{\alpha}_j^2}\right), k_3 = diag\left(\frac{1}{\hat{\phi}\hat{\alpha}_j^2}\right), k_4 = diag\left(\frac{p}{\hat{\phi}\hat{\alpha}_j^2}\right), k_5 = diag\left(p\sqrt{\frac{1}{\hat{\alpha}_j^2}}\right)$$

The five forms of matrix k_i above are alternative penalty values used in the Poisson Inverse Gaussian Generalized Ridge Estimator (PIGGRE) parameter estimation process. The selection of the k_i form aims to obtain stable parameter results and a

III. METHODS

The data used were obtained thru the official publication of the Central Bureau of Statistics of East Java (https://jatim.bps.go.id/id). The data includes 38 districts/cities in the province of East Java in the year 2024, consisting of the number of reported crimes (Y) expressed in cases, the percentage of the poor population (X_1) in percentages, the number of inhabitants (X_2) in persons, the average school duration (X_3) in years, the open unemployment rate (X_4) in percentages, the Gini index (X_5), and the Human Development Index (X_5). In this study, the methods used are the Poisson Inverse Gaussian Maximum Likelihood Estimator(PIGMLE), the Poisson Inverse Gaussian Ridge Regression Estimator(PIGRRE), and the Poisson Inverse Gaussian Generalized Ridge Estimator. In completing this research, the author used the software Rstudio. With the following steps:

1. descriptive statistical analysis
2. Poisson distribution test using the Kolmogorov-Smirnov test.
3. Overdispersion test to determine if there is overdispersion in the data.
4. Multicollinearity test using the Variance Inflation Factor (VIF).

5. Modeling using PIGMLE, PIGRRE, and PIGGRE.
6. Selection of the best model based on the smallest Mean Squared Error (MSE) value.
7. Parameter test on the best model: Simultaneous test (Likelihood Ratio Test), Partial test (Wald Test).

IV. RESULTS AND DISCUSSION

To compare the performance of the Poisson Inverse Gaussian Maximum Likelihood Estimator (PIGMLE), Poisson Inverse Gaussian Ridge Regression Estimator (PIGRRE), and Poisson Inverse Gaussian Generalized Ridge Estimator (PIGGRE) in handling overdispersion and multicollinearity in Poisson Inverse Gaussian regression models, a descriptive analysis of crime data at the regional level was first conducted. This analysis aims to obtain an overview of the characteristics of the data used. The results of the descriptive statistical analysis are presented in Table 1.

Table 1. Descriptive Statistical

Variabel	Min	Max	Mean	Median	SD
Y	105	5543	750.84	453.5	967.03
X_1	3.06	20.83	9.78	9.215	4.21
X_2	154687	2921696	1100737	1108.184	686331
X_3	5	12	8.5	8	1.70
X_4	1.56	6.49	4.04	4.075	1.18
X_5	0.233	0.435	0.332	0.329	0.037
X_6	66.72	84.69	75.31	74.555	4.76

Based on Table 1, the number of reported crimes (Y) shows considerable variation between regions, as indicated by the relatively high standard deviation compared to the mean value. This indicates significant differences in crime rates between regions. The variables of percentage of poor population, population size, average length of schooling, open unemployment rate, Gini index, and Human Development Index also show diverse data variations, reflecting differences in social and economic conditions between regions.

The data used will be examined to determine whether the dependent variable (Y) is Poisson distributed. This test is performed using the Kolmogorov-Smirnov test with the following results:

Table 2. Kolmogorov-Smirnov Test

Kolmogorov-Smirnov	
D	0.17732
P-value	0.1832

The test results in the table show that the value of $D_{hit} = 0.17732$ and the $p - value = 0.1832$. Because $D_{hit} = 0.17732$ and the p-value > $\alpha = 0.05$, H_0 is not rejected. Therefore, it can be concluded that the data is Poisson-distributed. Before the appropriate regression model is determined, an over dispersion test is performed to assess whether the variance of the data is greater than the mean value.

Next, before determining the appropriate regression model, an over dispersion test will be conducted to determine whether the data variance is greater than the average or mean value. The test result are presented in table 3.

Table 3. Over dispersion Test

Test Statistic	Value	p-value
z	3.8792	0.00005241
Dispersion	185.6322	-

At a significance level of 5%, the test results showed that the p-value was less than 0.05. Therefore, the H_0 is rejected, and it can be concluded that over dispersion is occurring in the data.

Multicollinearity is performed to determine if there is a strong relationship between the independent variables in the model. The test was conducted using the Variance Inflation Factor (VIF) value, with the condition that a VIF > 10 indicates high multicollinearity, while a VIF < 10 suggests that there are no serious multicollinearity issues.

Table 4. Multicollinearity Test

Variable Independent	Nilai VIF
X_1	3.92
X_2	1.54
X_3	21.32
X_4	1.65
X_5	1.60
X_6	17.57

Based on the Table 4, it can be seen that the variable Average School Duration (X_3) has a VIF value of 21.32 and the Human Development Index (X_6) has a value of 17.57. Both have a VIF value > 10, indicating high multicollinearity. Meanwhile, the other variables have a VIF value of less than 10, so it can be concluded that there are no multicollinearity issues with these variables. Poisson Inverse Gaussian Maximum Likelihood Estimator, Poisson Inverse Gaussian Ridge Regression Estimator, and Poisson Inverse Gaussian Generalized Ridge Estimator. To obtain more accurate and stable estimation results, modeling is performed using the Poisson Inverse Gaussian (PIG) with three approaches: MLE, Ridge Regression Estimator, and Generalized Ridge Estimator. For the Ridge Regression Estimator and Generalized Ridge Estimator, Ridge parameters k1-k5 were used with the results shown in Table 5.

Table 5. Parameter Estimates and MSE of PIGMLE, PIGRRE, and PIGGRE

Estimates	Parameter	β_0	β_1	β_2	β_3	β_4	β_5	β_6	MSE
PIGMLE	-	6.3255	0.1422	0.5868	-0.5348	0.1479	0.2805	0.5774	173412
PIGRRE	k_1	6.3247	0.1423	0.5870	-0.5322	0.1422	0.2804	0.5749	5.8687
	k_2	6.3239	0.1424	0.5871	-0.5296	0.1479	0.2804	0.5725	5.8664
	k_3	6.3239	0.1424	0.5871	-0.5297	0.1478	0.2804	0.5726	5.8665
	k_4	6.1673	0.1382	0.5913	-0.2784	0.1481	0.2662	0.3258	5.5259
	k_5	6.2038	0.1413	0.5928	-0.3122	0.1478	0.2697	0.3609	5.5973
PIGGRE	k_1	1.8813	0.1441	0.6677	-0.1467	0.0009	0.2458	0.3042	0.5972
	k_2	0.9112	0.1119	0.6682	-0.0626	0.0004	0.2065	0.2138	0.1974
	k_3	3.4213	0.1527	0.6592	-0.2703	0.0025	0.2674	0.4201	1.7834
	k_4	4.7295	0.1494	0.6498	-0.3672	0.0062	0.2755	0.5051	3.3255
	k_5	3.3418	0.1528	0.6597	-0.2642	0.0024	0.2667	0.4146	1.7056

Based on the analysis in the table, significant differences were found in terms of parameters and the Mean Square Error (MSE) value. For the Poisson Inverse Gaussian Maximum Likelihood Estimator model, the following values were obtained: $\beta_0 = 6.3255$, and the regression coefficients for the independent variables are $\beta_1 = 0.1422$, $\beta_3 = -0.5348$, $\beta_4 = 0.1479$, $\beta_5 = 0.2805$ and $\beta_6 = 0.5774$. The Mean Squared Error (MSE) value was 173412, indicating a very large prediction error. The coefficient values show that the variables X_1, X_2, X_4, X_5 and X_6 have a positive influence on the variable Y (number of reported crimes). This means that if one of these variables increases (with the other variables held constant), the number of reported crimes will also increase. And conversely, the variable X_3 has a negative influence on the variable Y, which means that if this variable increases (with the others remaining constant), then variable Y will decrease. However, because the Mean Squared Error (MSE) is high, indicating that the Poisson Inverse Gaussian Maximum Likelihood Estimation model is not accurate enough in forming the model, and also the difference in coefficient values between variables is quite large, and there was a fairly high degree of multicollinearity between the variables during the test, the estimation results are inefficient. Next, there is the Poisson Inverse Gaussian Ridge Regression Estimation (PIGRRE) model. The estimation results show that the PIGRRE estimation values are much smaller than those of PIGMLE, ranging from 5.8687 to 5.5259. This shows that the application of the ridge parameter is successful in improving model fitting, as the Mean Squared Error (MSE) decreases. Meanwhile, the Generalized Ridge Estimator of the Poisson Inverse Gaussian-Ridge Regression Estimator model showed better results than the two preceding models. The Mean Square Error (MSE) for the Poisson Inverse Gaussian Generalized Ridge Estimator (PIGGRE) model with Ridge parameters ranging from k_1 to k_5 was between 0.19 and 3.32. This value was the lowest of the other two models, indicating that the PIGGRE model produces the most accurate and stable results. In other words, this method can solve the problem of multicollinearity and produce more stable regression coefficients.

When selecting the best model. The method used is comparing the Mean Square Error (MSE) values. The smaller the MSE value, the better the model used. Below are the MSE values for different estimation methods.

Table 6. Comparison of Mean Squared Error (MSE) for PIGMLE, PIGRRE, and PIGGRE Methods

Methods	MSE
PIGMLE	173412
PIGRRE (k_1)	5.8687
PIGRRE (k_2)	5.8664
PIGRRE (k_3)	5.8665
PIGRRE (k_4)	5.5259
PIGRRE (k_5)	5.5973
PIGGRE (k_1)	0.5972
PIGGRE (k_2)	0.1974
PIGGRE (k_3)	1.7834
PIGGRE (k_4)	3.3255
PIGGRE (k_5)	1.7056

Given the smallest MSE value, found with the Poisson Inverse Gaussian Generalized Ridge Estimator (PIGGRE) with a ridge parameter k_2 of 0.1974. Therefore, it can be concluded that the use of the PIGGRE method is better at addressing multicollinearity in the case of the number of reported crimes in East Java Province in 2024. And thus, the following regression equation was obtained :

$$\log(\mu) = 0.9112 + 0.1119X_{1i} + 0.6682X_{2i} - 0.0626X_{3i} + 0.0004X_{4i} + 0.2065X_{5i} + 0.2138X_{6i}$$

Based on the Poisson Inverse Gaussian regression model formed above, it shows that socioeconomic variables have different effects on the number of reported crimes. The intercept value of exp (0.9112) = 2.487 indicates that when all independent variables are constant, the number of reported crimes will increase by 2.487. The number of reported crimes will increase by exp (0.1119) = 1.1184 if the poverty rate increases by 1 unit, assuming that other variables remain constant, in line with the findings of Soraya, et al. (2024), which state that poverty has a positive effect on crime in Indonesia. Furthermore, if the population increases by 1 unit, the number of reported crimes will increase by exp (0.6682)=1.950, assuming other variables remain constant, consistent with the research by Anozi and Novianda (2023). There is a decrease of exp (0.0626)=1.065 in the number of reported crimes if the average length of schooling increases by 1 unit and other variables are held constant, in line with the research by Amelia and Ridwan (2024) and Winda and Sentosa (2021), which shows that education has a negative effect on crime. There is an increase of exp(0.0004)=1.00004 in the number of reported crimes if the open unemployment rate increases by 1 unit and other variables are assumed to remain constant, as also found in the research by Soraya, et al. (2024). If the indexmes, and the number of inhabitants (X_2) has the greatest influence on the increase in the number of incidents. After the best model is obtained, a simultaneous significance test is performed to determine if the independent variables collectively influence the dependent variable.

Table 7. Simultaneous Significance Test

Test Statistic	Value
Nilai Likelihood Ratio (G)	32.2196
Derajat Bebas (df)	6
p-value	0.0000148

At a significance level of α = 0.05, the likelihood ratio test yields a test statistic value of G = 32.2196 with a p-value of 0.0000148. The critical value of the chi-square distribution with 6 degrees of freedom is 12.592. Since the value of G is greater than the critical value and the p-value is less than 0.05, the null hypothesis is rejected. Therefore, it can be concluded that the independent variables jointly have a significant effect on the dependent variable.

After establishing that the model is statistically significant based on the simultaneous test, a partial parameter test is performed to identify which independent variables have a significant individual effect on the dependent variable.

Table 8. Partial Parameter Test

Parameter	Estimate	Z-value	p-value	Decision
Intercept	0.9112	38.99	0.0000	Significant
a percentage of the poor population	0.1119	0.52	0.605	Not significant
population	0.6682	3.96	0.00007	Significant
the old school rate	-0.0626	-0.47	0.641	Not significant
open unemployment rate	0.00036	0.67	0.501	Not significant
Gini index	0.2065	1.35	0.176	Not significant
Human Development Index	0.2138	0.95	0.341	Not significant

At a significance level of $\alpha = 0.05$, the Wald test results show that variable X_2 has a Z value of 3.96 with a p-value less than 0.05. Since the Z value is greater than the chi-square critical value of 3.84, the null hypothesis is rejected.

V. CONCLUSIONS

Based on the analysis performed on the Poisson Inverse Gaussian distribution method, the Generalized Ridge Estimator is the best estimator for dealing with multicollinearity. This can be seen from the ridge-k parameter applied to the number of crimes reported in East Java province in 2024, which can be observed using the minimum Mean Square Error (MSE) value. The minimum Mean Square Error (MSE) value was obtained at the ridge k_2 parameter, which was 0.1974.

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